Code No: R161202 **R16** SET - 1

I B. Tech II Semester Regular Examinations, April/May - 2017
MATHEMATICS-II (MM)
(Com. to CE, EEE, ME, CHEM, AE, BIO, AME, MM, PE, PCE, MET)

Time: 3 hours Max. Marks: 70

Note: 1. Question Paper consists of two parts (**Part-A** and **Part-B)**
2. Answer **ALL** the question in **Part-A**
3. Answer any **FOUR** Questions from **Part-B**

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PART –A

1. a)	Explain the Bisection method.	(2M)
b)	Prove that= $E - 1$.	(2M)
c)	Write Newton's forward interpolation formula.	(2M)
d)	Write Trapezoidal rule and Simpson's 3/8th rule.	(2M)
e)	Write the Fourier series for $f(x)$ in the interval $(0, 2\pi)$.	(2M)
f)	Write One dimensional wave equation with boundary and initial conditions.	(2M)
g)	If $F(s)$ is the complex Fourier transform of $f(x)$, then prove that $F\{f(ax)\} = \frac{1}{a} F \frac{s}{a}$.	(2M)

PART –B

2. a)	Using bisection method, obtain an approximate root of the equation $x^3 - x - 1 = 0$.	(7M)
b)	Develop an Iterative formula to find the square root of a positive number N using Newton-Raphson method.	(7M)

3. a)	Evaluate $^{2}\left(\tan^{-1} x\right)$.	(6M)
b)	Using Newton's forward formula, find the value of $f(1.6)$, if	(8M)

x	1	1.4	1.8	2.2
$f(x)$	3.49	4.82	5.96	6.5
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4. a) Compute the value of $\int_{0.2}^{1.4} (\sin x - \log x + e^{x})\, dx$ using Simpson's $\frac{3}{8}$ th rule. (7M)

b) Using the fourth order Runge – Kutta formula, find $y(0.2)$ and $y(0.4)$ given that (7M)

$\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$, $y(0) = 1$.

5. a) Find a Fourier series to represent $f(x) = x - x^2$ in $-\pi \le x \le \pi$. Hence show that (7M)

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots\cdots = \frac{\pi^2}{12}.$$

b) Obtain the half range sine series for $f(x) = e^{x}$ in $0 < x < 1$. (7M)

6. a) Solve by the method of separation of variables (7M)

$4u_x + u_y = 3u$ and $u(0, y) = e^{-5y}$.

b) A tightly stretched string with fixed end points $x = 0$ and $x = L$ is initially in a (7M) position given by $y = y_0 \sin^3 \frac{\pi x}{L}$ if it is released from rest from this position, find the displacement $y(x, t)$.

7. a) Express the function $f(x) = \begin{cases} 1 & \text{for } |x| \le 1 \\ 0 & \text{for } |x| \ge 1 \end{cases}$ as a Fourier integral. Hence (7M) evaluate $\int_0^{\infty} \frac{\sin\lambda \cos\lambda x}{\lambda}\, d\lambda$.

b) Find the Fourier transform of $f(x) = \begin{cases} 1 - x^2 & \text{for } |x| \le 1 \\ 0 & \text{for } |x| > 1 \end{cases}$ hence evaluate (7M)

$$\int_0^{\infty} \frac{x\cos x - \sin x}{x^3} \cos\frac{x}{2}\, dx.$$

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PART –A

1. a) Explain the Method of false position. (2M)
b) Prove that $\nabla = 1 - E^{-1}$. (2M)
c) Write Newton’s backward interpolation formula. (2M)
d) Write Simpson’s 1/3rd and 3/8th rule. (2M)
e) Write the Fourier series for $f(x)$ in the interval $(0, 2L)$. (2M)
f) Write the suitable solution of one dimensional wave equation. (2M)
g) If $F(s)$ is the complex Fourier transform of $f(x)$, then prove that (2M)
$F\{f(x-a)\} = e^{ias} F(s)$.

PART -B

2. a) Using bisection method, compute the real root of the equation $x^3 - 4x + 1 = 0$. (7M)
b) Develop an Iterative formula to find the cube root of a positive number N using Newton-Raphson method. (7M)

3. a) Evaluate $\left(e^{x} \log 2x\right)$. (6M)
b) Using Newton’s forward formula compute $f(142)$ from the following table: (8M)

x	140	150	160	170	180
$f(x)$	3.685	4.854	6.302	8.076	10.225

4. a) Evaluate, $\int_{0}^{2} e^{-x^2} dx$ by using Trapezoidal rule and Simpson’s $\frac{1}{3}$rd rule taking $h = 0.25$. (7M)
b) Find the value of y at $x = 0.1$ by Picard’s method, given that (7M)
$\frac{dy}{dx} = \frac{y-x}{y+x}$, $y(0) = 1$.
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5\. a) Given that $f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$. Find the Fourier series for $f(x)$. (7M)

Also deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots\cdots = \frac{\pi^2}{8}$.

b) Express $f(x) = x$ as a half-range cosine series in $0 < x < 2$. (7M)

6\. a) Solve by the method of separation of variables (7M)

$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$, given $u(x, 0) = 6e^{-3x}$.

b) A string of length L is initially at rest in equilibrium position and each of its points (7M) is given the velocity $\left(\frac{\partial y}{\partial t}\right)_{t=0} = b\sin^3\frac{\pi x}{L}$. Find displacement $y(x, t)$.

7\. a) Express $f(x) = \begin{cases} 1 & for\ \ 0 \le x \le \pi \\ 0 & for\ \ x > \pi \end{cases}$ as a Fourier sine integral and hence evaluate (7M)

$\int_0^\infty \frac{1 - \cos(\pi\lambda)}{\lambda}\sin(x\lambda)\, d\lambda$.

b) Find the Fourier sine and cosine transform (7M)
of $f(x) = e^{-ax}$, $a > 0, x > 0$.

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PART –A

1. a) Explain the Newton-Raphson method. (2M)
   b) Prove that $\delta = E^{1/2} - E^{-1/2}$. (2M)
   c) Write Lagrange's interpolation formula for unequal intervals. (2M)
   d) Explain Taylor's series method for solving IVP $\frac{dy}{dx} = f(x,y)$ with $y(x_0) = y_0$. (2M)
   e) Write the Fourier series for $f(x)$ in the interval $(-\pi, \pi)$. (2M)
   f) Write the suitable solution of one dimensional heat equation. (2M)
   g) If $F(s)$ is the complex Fourier transform of $f(x)$, then prove that (2M)
   $F\{f(x)\cos ax\} = \frac{1}{2}\left[F(s+a) + F(s-a)\right]$.

PART –B

2. a) Using Regula-Falsi method, compute the real root of the equation $x^3 - 4x - 9 = 0$. (7M)
   b) Develop an Iterative formula to find $\frac{1}{N}$. Using Newton-Raphson method. (7M)

3. a) Evaluate $\frac{x^2}{\cos 2x}$. (6M)
   b) Compute $f(27)$ Using Lagrange's formula from the following table: (8M)

x	14	17	31	35
$f(x)$	68.7	64.0	44.0	39.1
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4. a) Evaluate $\int_0^{0.6} e^{-x^2}\,dx$ by using Simpson's $\frac{1}{3}$ rd rule taking seven ordinates. (7M)

b) Given that $\frac{dy}{dx} = 2 + \sqrt{xy}$, $y(1) = 1$. (7M)

Find $y(2)$ in steps of **0.2** using the Euler's method.

5. a) Find the Fourier series for the function $f(x) = \begin{cases} x, & 0 \le x \le \pi \\ 2\pi - x, & \pi \le x \le 2\pi \end{cases}$. (7M)

Also deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots\cdots = \frac{\pi^2}{8}$.

b) Obtain the Fourier expansion of $f(x) = x \sin x$ as a cosine series in $(0, \pi)$. (7M)

6. Solve the Laplace's equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ in a rectangle in the xy-plane, (14M)
$0 \le x \le a$ *and* $0 \le y \le b$ satisfying the following boundary condition
$u(0, y) = 0$, $u(a, y) = 0$, $u(x, b) = 0$ *and* $u(x, 0) = f(x)$.

7. a) Find the Fourier sine transform of the function (7M)

$$f(x) = \begin{cases} x, & 0 < x < 1 \\ 2 - x, & 1 < x < 2 \\ 0, & x > 2 \end{cases}.$$

b) Find the Fourier cosine integral and Fourier sine integral of (7M)
$f(x) = e^{-kx}$, $k > 0$.

Code No: R161202 **R16** SET - 4

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PART –A

1. a) Explain Iteration method. (2M)
b) Prove that $\propto = \frac{1}{2}\left(E^{1\ 2} + E^{-1\ 2}\right)$. (2M)
c) Prove that $^3 y_2 = \nabla^3 y_5$. (2M)
d) Explain Runge-Kutta method of fourth order for solving IVP $\frac{dy}{} = f(x, y)$ with $y(x_0) = y_0$. (2M)
e) Write the Fourier series for $f(x)$ in the interval $(-L, L)$. (2M)
f) Write the various possible solutions of two-dimensional Laplace equation. (2M)
g) If $F(s)$ and $G(s)$ are the complex Fourier transform of $f(x)$ and $g(x)$, then prove that $F\{a f(x) + b g(x)\} = a F(s) + b G(s)$. (2M)

PART –B

2. a) Find a positive real root of the equation $x^4 - x - 10 = 0$ using Newton-Raphson's method. (7M)
b) Explain the bisection method. (7M)

3. a) Evaluate $^2(\cos 2x)$. (6M)
b) Using Newton's backward formula compute $f(84)$ from the following table: (8M)

x	40	50	60	70	80	90
$f(x)$	184	204	226	250	276	304
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4\. a) Evaluate $\int_0^1 e^{-x^2}\,dx$ by using Trapezoidal rule with $n = 10$. (7M)

b) Obtain Picard's second approximate solution of the initial value problem (7M)

$$\frac{dy}{dx} = \frac{x^2}{y^2+1},\ y(0) = 0 .$$

5\. a) Obtain the Fourier series $f(x) = \left(\frac{\pi - x}{2}\right)^2$ in the interval $0 < x < 2\pi$. Deduce that (8M)

$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots\cdots = \frac{\pi^2}{6} .$$

b) Express $f(x) = x$ as a half-range cosine series in $0 < x < 2$. (6M)

6\. a) Solve by the method of separation of variables (7M)

$$\frac{\partial u}{\partial x} = 4\frac{\partial u}{\partial y} \ \ and \ \ u(0, y) = 8e^{-3y} .$$

b) Solve the Laplace's equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ in a rectangle in the xy-plane, $0 \le x \le a$ and $0 \le y \le b$ satisfying the followingboundary condition $u(x, 0) = 0$, $u(x, b) = 0$, $u(0, y) = 0$ and $u(a, y) = f(y)$. (7M)

7\. a) Find the Fourier cosine integral and Fourier sine integral of (7M)

$$f(x) = e^{-ax} - e^{-bx},\ a > 0, b > 0.$$

b) Find the Fourier transform of $e^{-a^2x^2}$, $a > 0$. Hence deduce that $e^{-\frac{x^2}{2}}$ is self reciprocal in respect of Fourier transform. (7M)